NOTATION

ρ, z, dimensionless coordinates; Θ, dimensionless temperature; Q, dimensionless volume heat-liberation density per unit time; $Fo = \varkappa\tau/\delta^2$, Fourier number; $Bi_1(\rho, Fo) = \alpha(\rho, Fo) \cdot \delta/\lambda$, Biot number; $\varkappa$, thermal diffusivity coefficient; δ, plate thickness; τ, time; $\alpha(\rho, Fo)$, heat-liberation coefficient; λ, thermal conductivity coefficient; i, summation index; J_0, zero order Bessel function of the first kind.

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HEAT TRANSFER OF A SEMITRANSPARENT PLATE UNDER CONDITIONS OF A REGULAR REGIME OF THE SECOND KIND

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UDC 536.33:536.241

The kinetics of the cooling of a plane semitransparent layer under conditions of a regular regime of the second kind is analyzed.

Heating and cooling of optical blanks made of glass and ceramics quite often are carried out under conditions of linearly varying ambient temperature. In this case the calculation of the rate of cooling and the temperature drops in the glass, in practice, is carried out according to empirical relations [1, 2] or according to dependencies of a regular regime of the second kind [3]. Moreover, it is known that at sufficiently high temperatures in glass and diathermic ceramics, bulk self-radiation of the material develops, and use of the ordinary Fourier equation is not very effective here. The features of the heating of a semitransparent layer under conditions of a regular regime of the first kind were investigated in [4].

Study [5] is the only work in which an analysis is given of the features of a regular regime of the second kind in semitransparent materials on the basis of the exact equations of radiant—conductive heat exchange. However, we should note that an analysis was performed for a narrow range of parameters of the regime and at low temperatures, when the radiant component of heat exchange appeared weak. Therefore, the conclusion of the authors concerning the existence of a regular regime of the second kind in semitransparent materials, analogous to the situation in the classical theory of heat conduction, has little substantiation.

The present study is devoted to a theoretical study of the features of the unsteady process of radiation—conduction heat exchange under conditions of a regular regime of the second kind, when the radiation component makes a noticeable contribution to the value of the total heat flux.

We consider the problem in the following formulation. A plate of glass of thickness 2d with optically smooth surfaces, the transfer of heat within which occurs simultaneously by

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radiation and by heat conduction, at some initial time $\tau = 0$ has the uniform temperature distribution $T(0, x) = T^\circ$. Then the plate is placed in a medium whose temperature varies according to the linear law $T_e = T^\circ - b\tau$, and the coefficient of convective heat exchange is given and is constant in time. The thermophysical properties of the material K and $C\gamma$ and its spectral radiation characteristics n and α are assumed to be known and independent of the temperature. The sample of glass is surrounded by bodies whose degree of blackness is close to unity. The bodies are subjected to forced cooling, i.e., the sample is surrounded by a radiation cooler. We should also add that the ratio of the surfaces of the bodies participating in the radiation heat exchange to the surface of the semitransparent plate is much greater than unity.

It is required to find the temperature distribution and the heat fluxes at subsequent times.

The temperature field for a combined mechanism of transfer of heat by radiation and by heat conduction is described by the equation of radiative–conductive heat exchange [6]

$$\operatorname{div}(-K \operatorname{grad} T + \overline{E}) = -C\gamma \frac{\partial T}{\partial \tau}, \tag{1}$$

where $\overline{E}$ is the vector of the thermal-radiation flux; since it is a nonlinear functional of the temperature field, it depends on the nature of the reflection at the boundaries, the spectral dependence of the absorption coefficient of the material, and the thickness of the sample:

$$E = \pi \int_{(\lambda_t)} \int_0^{\pi/2} (\Phi^+ + \Phi^-) \sin 2\Theta d\Theta d\lambda. \tag{2}$$

The distribution of the intensities Φ^+ and Φ^- over the thickness of the layer for all necessary values of the spectral and angular variables is described by a transport equation for purely absorbing media

$$\begin{gathered} \cos\Theta \frac{\partial \Phi^+}{\partial x} = -\alpha\Phi^+ + \frac{\alpha n^2}{\pi} B(\lambda, T), \\ -\cos\Theta \frac{\partial \Phi^-}{\partial x} = -\alpha\Phi^- + \frac{\alpha n^2}{\pi} B(\lambda, T). \end{gathered} \tag{3}$$

We close Eqs. (1) and (3) by means of the following boundary conditions: for the radiative component of heat transfer [7]

$$\Phi^+ - R\Phi^- = 0, \quad x = 0, \quad \Phi^- - R\Phi^+ = 0, \quad x = 2d \tag{4}$$

and for the conductive component

$$\begin{gathered} K \frac{\partial T}{\partial x} + h_e (T_e - T) - \int_{(\lambda_{op})} \varepsilon_\lambda B(\lambda, T) d\lambda = 0, \quad x = 0, \\ -K \frac{\partial T}{\partial x} + h_e (T_e - T) - \int_{(\lambda_{op})} \varepsilon_\lambda B(\lambda, T) d\lambda = 0, \quad x = 2d. \end{gathered} \tag{5}$$

In the boundary conditions (4), the second term takes into account the effects of multiple partial and total internal reflection of the bulk self-radiation of the substance on the surfaces of the layer. Conditions (4) are written under the assumption that the external fluxes of the thermal radiation are absent. This is valid for those cases in which the optical thickness of the layer of the gas washing the sample is small, and the surrounding bodies are subjected to forced cooling, completely absorbing the self-radiation of the glass plate. The third term of the boundary conditions (5) takes into account the self-radiation of the surfaces.

To find the spectral directional radiating power of the plate Φ_e^+ and Φ_e^- we use the conditions

$$\begin{gathered} \Phi_e^+ - n^{-2}(1 - R)\Phi^+ = 0, \quad x = 2d, \\ \Phi_e^- - n^{-2}(1 - R)\Phi^- = 0, \quad x = 0. \end{gathered} \tag{6}$$

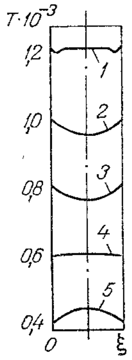

Fig. 1

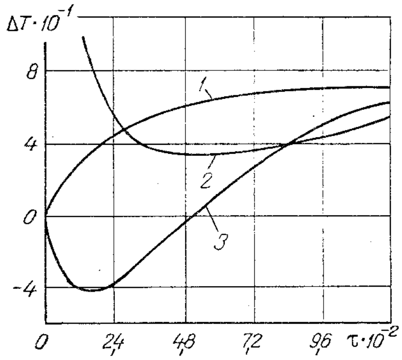

Fig. 2

Fig. 1. Distribution of temperature T (°C) along the thickness of a semitransparent layer $\xi = x/d$ for various times (sec): 1) $\tau = 2.4$; 2) 60; 3) 240; 4) 540; and 5) 780.

Fig. 2. Dependence of temperature drop ΔT (°C) on time τ (sec) for three different cases: 1) opaque sample for $\varepsilon = 0$; 2) the same, for $\varepsilon = 1$; 3) semitransparent sample.

In Eqs. (6) it is necessary to take into account that the wavelength of the radiation leaving the plate and the wavelength of the radiation in the medium are related by the equation $\lambda_m = \lambda/n$, where λ_m is the wavelength in the substance.

The formulated boundary-value problem (1)-(6) was solved in finite differences. We used a nonexplicit difference scheme with a uniform grid with respect to coordinate and time. The algorithm for solution of the finite-difference equations was similar to the known procedure of factorization and was realized sufficiently simply using a computer. The general scheme and the detailed algorithm of solution are presented in [7, 8].

We turn to a discussion of the results of the numerical analysis. Calculations were carried out for a material whose absorption spectrum is represented by a single-band model with transmission window $\alpha = 50\ m^{-1}$ for a spectral interval $\lambda_t = 0.25$–4.8 μm. The thermal radiation characteristics needed for the calculation were assumed to be equal to $n = 1.5$, $\varepsilon = 0$, the thermophysical properties $K = 1.582$ W/m•°C, $C\gamma = 2.5$ J/m^3, the thickness of the layer was $2d = 2 \cdot 10^{-2}$ m.

As a result of the solution, we determined the heat transfer of a semitransparent plate in the surrounding medium owing to various mechanisms of transfer during the cooling process. The purpose is to study the effect of the bulk self-radiation of the substance on the dynamics of the temperature field.

The temperature distribution over the thickness for various times is represented in Fig. 1. For the case of pure heat conduction after the passage of a certain time $Fo \geqslant 0.5$, there approaches a quasistationary regime, the differing feature of which is the equivalence of the change of the temperature profile at successive moments of time. For a semitransparent plate, nothing similar is observed. For the first seconds, near the surfaces being cooled, the temperature profile has minima, which during the cooling process gradually withdraw from the surfaces to the center of the plate (Fig. 1, curve 1, $\tau_1 = 2.4$ sec). After the points of the minimum reach the center of the sample, the temperature profile during a sufficiently continuous interval of time maintains the shape of a parabola with minimum at point $x = d$ (curves 2 and 3, $\tau_2 = 60$ sec, $\tau_3 = 240$ sec). In this case the nature of the variation of temperature of various points of the sample continues to remain unequal: the most intense cooling is undergone by the surface layers of the material and there is a gradual equalization of the temperature over the thickness of the layer (curve 4, $\tau_4 = 540$ sec). In proportion to the further cooling of the plate, the temperature gradients on the surfaces change to the opposite sign (curve 5, $\tau_5 = 780$ sec). Only for this final stage of cooling is there observed a tendency toward the regularization of the temperature field.

The time dependence of the temperature drop $\Delta T = T(d) - T(0)$ between the center of the plate and its surface is shown in Fig. 2. The curves in the figure are obtained with account of various factors. Curve 3 illustrates the nature of the variation of the function $\Delta T(\tau)$ for a partially transparent material, and curves 1 and 2 correspond to temperature

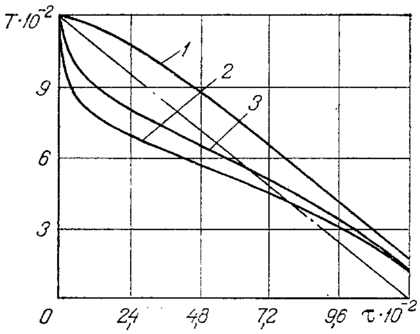

Fig. 3

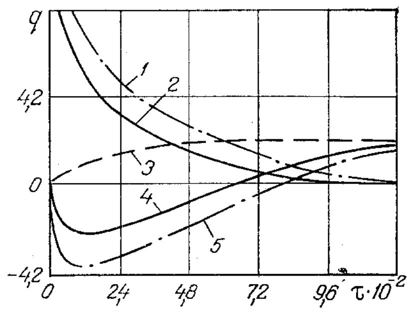

Fig. 4

Fig. 3. Variation of temperature of the surfaces of the plate T in time τ for three different cases: 1) pure heat conduction for $\varepsilon = 0$; 2) the same, for $\varepsilon = 1$; 3) combined heat transfer by radiation and heat conduction; dot–dash curve) ambient temperature.

Fig. 4. Dependence of convective and radiative thermal fluxes q (W/cm^2), exiting through one of the surfaces of the layer, on the time τ (sec) for three different cases: 1, 2) radiative thermal flux; 3, 4, 5) convective flux; 2, 4) combined heat transfer; 1,5) pure heat conduction for $\varepsilon = 1$; 3) the same, for $\varepsilon = 0$.

drops $\Delta T(\tau)$ in the limiting cases of nontransparent (opaque) samples, having differing values of degree of blackness of the surfaces, with other parameters remaining equal. In the case $\varepsilon = 1$ (curve 2), the temperature drop in the layer rapidly attains a maximum ($\Delta T_{max} = 253°C$ for $\tau = 20$ sec), then decreases to a minimum ($\Delta T_{min} = 34°C$ for $\tau = 540$ sec), asymptotically approaching ΔT^*, which is observed for the case of pure heat conduction at $\varepsilon = 0$. The temperature drop in a semitransparent plate also has an extremal nature, but differs significantly from the case $\varepsilon = 1$ being considered. The bulk radiation of the substance leads to the fact that the quantity $\Delta T(\tau)$ is negative here, and then in proportion to the cooling of the sample, the dependence $\Delta T(\tau)$ passes through a minimum, beginning to increase subsequently, approaching the asymptote ΔT^*.

The explanation for such an unusual nature of cooling of a semitransparent plate should be sought in the interaction of the various forms of transfer of thermal energy. Here (from a comparison of the given curves) we can conditionally separate two stages. In the initial stage of cooling when the general level of temperature is sufficiently high, intense bulk excitation of the substance leads to the appearance in the sample of certain effective bulk discharges of energy, nonuniformly distributed over the thickness of the layer. Their intensity is such that the rate of cooling of different points proves to be greater than the rate of temperature variation of the surrounding medium. In this stage there is simultaneously radiative cooling of the layer and its convective heating. A clear representation of the rate of cooling of the surfaces yields Fig. 3, in which results are presented for the calculation of $T(\tau)$ for three different cases. In the second stage of cooling at $T \approx 500°C$, the intensity of bulk radiation of the substance drops noticeably, and the surface temperature becomes greater than the ambient temperature. In this stage, convective cooling of the semitransparent plate begins and subsequently predominates.

Results on heat exchange are represented in Fig. 4, in which we present radiative and convective heat fluxes on one of the surfaces of the layer for three different cases of heat exchange, where we assume as positive those fluxes that emanate from the plate. The solid lines characterize the results of calculation for a semitransparent plate; the dot–dash line represents the limiting case of an opaque plate with degree of blackness of surfaces $\varepsilon = 1$; the dashed line is the second limiting case of pure heat conduction of the material for $\varepsilon = 1$. Here, curves 3, 4, and 5 correspond to a convective flux, and curves 1 and 2 correspond to a radiative flux.

For finding the radiating power of the semitransparent layer we used the condition of balance of the radiant energy on the interior surface of the layer. All the thermal rays Φ^+

propagating in directions bounded by the range $0 \leqslant \Theta \leqslant \arcsin(1/n)$ on the interior boundary of the glass with air undergo partial reflection within the layer and, after being refracted, leave the plate. As a result of refraction by rays leaving the plate, there corresponds to this a different range of directions $0 < \Theta \leqslant \pi/2$, and their intensity will be $\Phi_e^+ = n^{-2}(1 - R)\Phi^+$. In this case one should remember the variation of wavelength of the thermal radiation.

According to what has been discussed above, the magnitude of the radiant flux from the surface of the layer x = d was determined by integration of the intensity Φ_e^+ over all directions and the entire range of partial transparency of the material:

$$q_{\text{rad}} = \frac{1}{n^2} \int\limits_{(\lambda_t)} \int\limits_0^{\pi/2} (1 - R)\, \Phi^+(2d) \sin\Theta \cos\Theta d\Theta d\lambda. \tag{7}$$

The numerical analysis that has been carried out shows that the features of the cooling of a semitransparent layer for radiative-conductive heat exchange differ considerably from the nature of the variation of the temperature field for pure molecular heat conduction under conditions of a regular regime of the second kind. In the physical relations, the features established are a consequence of the bulk radiation of the substance, which, in a substantially nonlinear way, depends on the temperature field in the plate. Thus, direct application of the separate positions of the classical theory of the regular regime of the second kind to semitransparent materials can lead to incorrect conclusions and errors.

NOTATION

τ, time; T, temperature; T_e, temperature of the surrounding medium (the ambient temperature); T°, initial temperature of the layer; h_e, coefficient of convective heat exchange; x, coordinate; α, coefficient of absorption of the substance; n, index of refraction; $B(\lambda, T)$, surface density of radiation of a black body; $C\gamma$, bulk specific heat; K, thermal conductivity; λ, wavelength of the radiation; λ_t, range of wavelengths in which the material is partially transparent; λ_{op}, range of wavelengths in which the material is opaque; ε, degree of blackness of the surfaces in the range of λ_{op}; $R(\Theta)$, coefficient of reflection from the inner surfaces of the layer; Φ^+, intensity of the rays consisting of acute angles with the inner normal to the surface x = 0; Φ^-, intensity of rays in the opposite direction; q, thermal flux; Θ, angle measured from the inner normal to the surface x = 0.

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